Sparse ℓ_1-ℓ_1 Multi-Tensor Imaging at the Price of DTI

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I. Introduction

Why use diffusion datasets to reconstruct the whole ODF if all we really need for some applications (ex. tractography) are the orientation of a few diffusion peaks? This question is even more important if we are limited to clinical applications for which only there is only time to acquire a diffusion tensor imaging (DTI)-like dataset (with 6 to 32 diffusion measurements). A more intuitive approach to make the best of the little data we have would be to fit a few one-peak model at each voxel. The diffusion tensor can be used as one such model [1]. In our technique, we then use a ℓ_1 data fitting term with a ℓ_1 sparsity regularization to fit the model to the signal. This ℓ_1-ℓ_1 is new in local diffusion estimation and allows us to use a general purpose linear program solver as opposed to the more common ℓ_2-ℓ_1 approach [3], [4].

II. Sparse Multi-Tensor Imaging

A. Problem statement

We want to approximate the diffusion signal S at q-points g_k with a sum of tensor T_i,

$$S_k = S_0 \sum_{i=1}^{M} f_i e^{-b g_k^t T_i g_k} + \epsilon,$$

where S_0 is the signal without diffusion weighting, f_i is the relative volume fraction of tensor T_i, M is the number of compartment, b is the diffusion sensitization strength and ϵ is the noise. We re-write our signal approximation as $S/S_0 = Uf + \epsilon$, where U is our dictionnary. We then search for a sparse and non-negative f^* so that Uf^* fits as best as possible the measured signal S. To do so, we solve the ℓ_1-ℓ_1 problem

$$f^* = \arg\min_{f \geq 0} \left[\|Uf - S\|_1 + \gamma \|f\|_1 \right],$$

where γ is the sparsity regularization constant used to adjust the trade-off between sparsity and data fitting.

B. Building the dictionnary

We build the dictionnary U from q-points g_i for $i = 1, 2, .., N$ uniformly spaced on one q-space shell (for one b-value), points p_j for $j = 1, 2, .., D$ evenly spaced on a half-sphere and tensor T. The j^{th} column of the dictionnary is the value of the diffusion defined by tensor T rotated at directions p_j at the q-points g_i i.e. $U_{ij} = e^{-b g_i^t T_j g_i}$, where T_j is T aligned to p_j.

C. Optimization

To solve the ℓ_1-ℓ_1 problem we re-casted it as a linear program [2] and used MATLAB's *LINPROG*. We used a fixed γ for the first pass and re-launched the optimization with different γ until f^* satisfied a norm concentration criterion

$$\frac{\|f_\alpha^*\|_1}{\|f^*\|_1} \geq \beta,$$

where $f_\alpha^* = Thresh_\alpha(f^*)$ (coefficients smaller than α are set to zero) and β is the minimum proportion of f^* ℓ_1-norm concentrated in it's few big coefficients i.e. the coefficients of f_α^*. Once f^* is determined, we first threshold it to remove the unwanted small components $f_\alpha^* = Thresh_\alpha(f^*)$. We then look at each maxima 2 by 2 and eliminate those with an angular distance smaller than δ as they are most likely representing the same diffusion peak. Sometimes the resulting vector $f_{\alpha,\delta}^*$ still has more peaks than a fixed μ, if so we remove the smallest intensity peak until we reach μ.

III. ISBI HARDI Contest

To fit the specific parameters of the contest, the maximum number of compartment μ was set to 3 and the tensor T used to build the dictionnary had eigenvalues $[\lambda_1\ \lambda_2\ \lambda_3] = [1.7\ 0.3\ 0.3] \times 10^{-3}$. The other parameters were tuned independently for each signal-to-noise ratio (SNR) (10, 20, 30) to first maximize the number of voxels where the right number of compartment was found and then to minimize the angular error. On the training data, we varied simulations with $N = 12, .., 50$ and b-values $= 500, .., 1500$. We found no significant amelioration of peak detection with high b-values while the angular error was affected by the SNR drop at higher b-values. The optimal trade-off between correct number of peaks and angle accuracy was obtained for low b-value and a low number of diffusion measurements. Therefore, the dataset asked for the ISBI contest was similar to a standard DTI with $N = 24$ diffusion measurements and $b = 750$ s/mm^2.

The final parameters were carefully set as $D = 500$ (dictionnary of size 500 tensors), initial $\gamma = 5$, $\beta = 0.5$, $\delta = 28.65°$ and $[\alpha_{10}\ \alpha_{20}\ \alpha_{30}] = [0.15\ 0.13\ 0.11]$. These parameters, especially D, δ and α have a huge impact on the quality of the reconstruction as they define the maximum angular resolution of the method while controling the over-fitting (big noise peak where a tensor is fitted) and the under-fitting (true small peak discarted as noise). To estimate the volume fraction, we used the values of $f_{\alpha,\delta}^*$ normalized to sum to one. Since we also needed an ODF estimation for the contest, we further refined our estimation by trying different combinations of $[\lambda_1\ \lambda_2\ \lambda_3]$ in the contest's range $\left(\lambda_1 \in [1,2] \times 10^{-3}, \lambda_2 = \lambda_3 \in [0.1, 0.6] \times 10^{-3}, FA \in [0.75, 0.9]\right)$ for each direction of $f_{\alpha,\delta}^*$ and chose the ones that minimized the residual with respect to the measured signal. We then used the analytical tensor ODF formula provided by the contest.

References

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