Compressive Sensing DSI

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***Abstract*—Compressive Sensing (CS) [2], [1] offers an efficient way to decrease the number of measurements required in Diffusion Spectrum Imaging (DSI). This method aims to reconstruct the Ensemble Average Propagator (EAP) and, for the purpose of this contest, we compute the numerical Orientation Distribution Function (ODF) by integrating the EAP over a solid angle. In this abstract, we briefly describe three important points underlying the CS technique in order to accelerate DSI, namely the sparsity, the Restricted Isometry Property (RIP) and the ℓ_1 reconstruction scheme. Due to the high b-values required in the sampling protocol, our approach enters the heavyweight sampling category. Nevertheless, only 64 measurements are used for the reconstruction.**

I. Sparse representation of the EAP

A sparse representation of the EAP is one of the key ingredient of an efficient CS recovery. In this work, we use the Discrete Wavelet Transform (DWT) based on the biorthogonal Cohen-Daubechies-Feauveau (CDF) 9-7 wavelet in order to sparsely describe the EAP. It has been shown that the CDF 9-7 wavelet leads to a very sparse representation of the EAP [4], [5]. In Fig. 1, we show the decomposition scaling and wavelet functions.

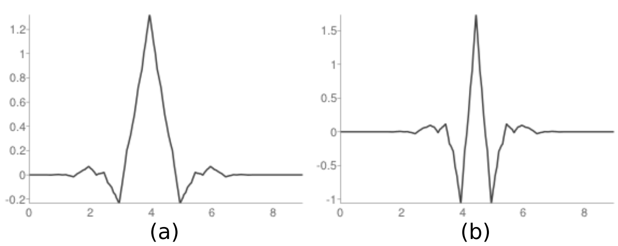

Fig. 1. The CDF 9-7 wavelet. (a) Decomposition scaling function. (b) Decomposition wavelet function.

II. The Restricted Isometry Property (RIP)

The RIP property is respected with a high probability when the acquisition are taken at random [1]. The possibility to acquire q-space samples in a random fashion is an important aspect in DSI that facilitates the application of the CS technique. In Fig. 2, we show an example of random sampling scheme.

In practice, since the diffusion signal is antipodally symmetric, we generate the sampling scheme in half the grid and then symmetrize it. For the contest we asked for 64 measurements in order to compare our results with HARDI-like reconstruction. However, because we require high b-values in the acquisition protocol we enter the heavyweight sampling category.

III. The ℓ_1 reconstruction scheme

The solution x of our problem is given by solving the following convex optimization problem [3], [1] :

$$argmin_x J(x) = \|TF_{u0}(x) - E_u\|_2^2 + \lambda\|\Psi x\|_1 \qquad (1)$$

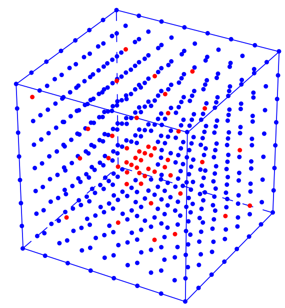

Fig. 2. 3D random sampling scheme. The blue points represent a full sampling of the 3D cartesian grid. The red points are samples taken at random.

The first term is the data consistency constraint, $\|\Psi x\|_1$ is the sparsity constraint with Ψ an operator representing the CDF 9-7 wavelet based DWT. λ is the Lagrange parameter that defines the confidence we put in the measured signal E_u. The data consistency constraint enables the solution to remain close to the raw data acquisition. TF_{u0} is the 3D undersampled Fourier operator defined by three operations. The first operation consists in applying a 3D Fourier transform. The latter is undersampled in a random manner. Then, the other coefficients are replaced by zero values. Hence, the acquired data are defined by $E_u = TF_{u0}(P)$ with P the propagator to be recovered. x is the estimated propagator so $TF_{u0}(x)$ is the undersampled Fourier transform of the estimated propagator. Equation (1) finds the sparsest solution with respect to Ψ that corresponds to the acquired data.

IV. Technical aspect regarding the ODF computation

In order to obtain the ODFs, we do not integrate between 0 and and R_{max} (the maximum radius of the EAP in the 3D cartesian grid). Instead, we integrate the EAPs between the range $[R_{\text{max}} \times \alpha, R_{\text{max}} \times \beta]$ with $\alpha, \beta \in [0, 1]$, because we observed that well chosen bounds for the radial integration lead to better results in terms of DNC and AE. Based on previous experiments, we choose $\alpha = 0.2$ and $\beta = 0.7$.

References

[1] E.J. Cands and M.B. Wakin. An introduction to compressive sampling. *Signal Processing Magazine, IEEE*, 25(2):2130, 2008.
[2] D.L. Donoho. Compressed sensing. *Information Theory, IEEE Transactions on*, 52(4):1289–1306, 2006.
[3] M. Lustig, D. Donoho, and J.M. Pauly. Sparse mri: The application of compressed sensing for rapid mr imaging. *Magnetic Resonance in Medicine*, 58(6):11821195, 2007.
[4] Sylvain Merlet, Michael Paquette, Rachid Deriche, and Maxime Descoteaux. Ensemble average propagator reconstruction via compressed sensing: Discrete or continuous bases ? In *ISMRM 20th Annual Meeting (2012)*, page 2277, Australie, May 2012.
[5] Etienne Saint-Amant and Maxime Descoteaux. Sparsity characterization of the diffusion propagator. In *Proceedings of ISMRM*, 2011.